Income Distribution and Macroeconomics

Oded Galor and Joseph Zeira

The Galor-Zeira Model

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- Production in the unskilled-intensive sector:

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- Unskilled labor:

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Factor Prices

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$$(r_t, w_t^s, w_t^u) = (r, w^s, w^u) \qquad \forall t$$

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- Differ in:

  Parental income $\Rightarrow$ Inv't in HC

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- Second period of life (Period $t+1$):
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Second period budget constraint:

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$\omega_{t+1} \equiv$ wealth in period $t+1$

Member of Generation t: Optimization

$$\{c_{t+1}, b_{t+1}\} = \arg\max[\alpha \ln c_{t+1} + (1-\alpha) \ln b_{t+1}]$$

$$\text{s.t.} \qquad c_{t+1} + b_{t+1} \leq \omega_{t+1}$$

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$\Longrightarrow$ v^t is monotonic increasing in 2nd period wealth, ω_{t+1}

$\Longrightarrow$ maximization of ω_{t+1}, is the basis of occupational choices

Fundamental Assumptions

- Imperfect Capital Markets:

$$r < i \tag{A1}$$

$r \equiv$ interest rate for lender

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- Fixed cost of education (Indivisibility of inv't in HC)

$$h > 0 \qquad \theta \in [0, 1] \tag{A2}$$

Income: Unskilled Individuals

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$$\omega_{t+1}^{u} \quad = \quad (w^{u}$$

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Income: Unskilled Individuals

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$$
\begin{aligned}
\omega_{t+1}^{u} &= (w^{u} + b_t)(1 + r) + w^{u} \\
&= w^{u}(2 + r)
\end{aligned}
$$

Income: Unskilled Individuals

$$\omega_{t+1}^{u} = (w^{u} + b_t)(1+r) + w^{u}$$
$$= w^{u}(2+r) + (1+r)b_t$$

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Assumptions

- Investment in human capital is *not* beneficial for individuals who must finance the entire cost of education via borrowing

$$w^s - (1+i)h < 0 \tag{A3}$$

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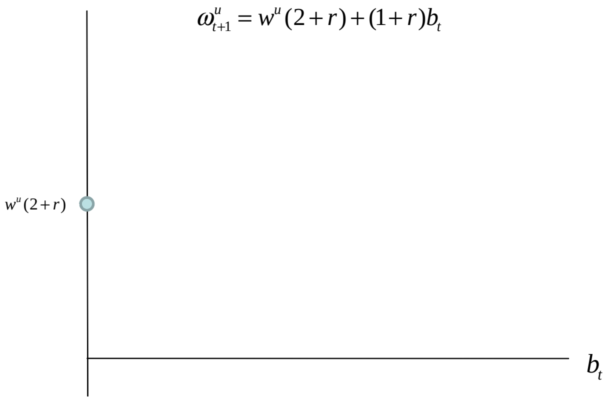
- Investment in human capital is beneficial for individuals who can finance the entire cost of education *without* borrowing

$$w^s - (1+r)h > w^u(2+r) \tag{A4}$$

Income from Being Unskilled Worker



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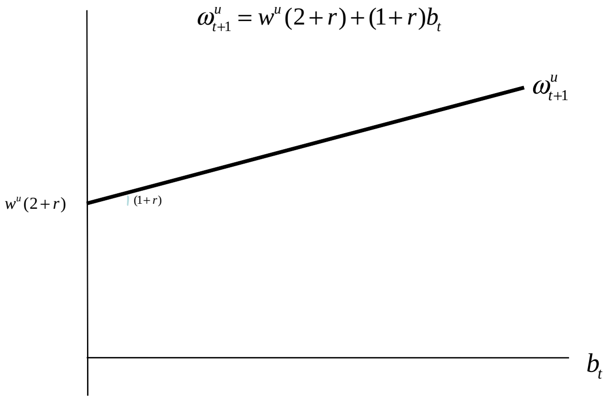
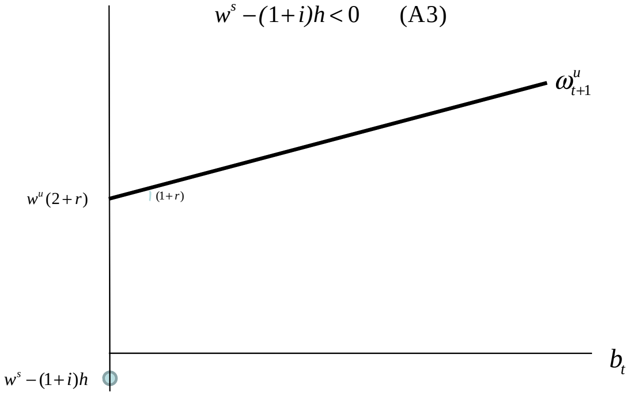


Income from Being Skilled Worker: Borrowers



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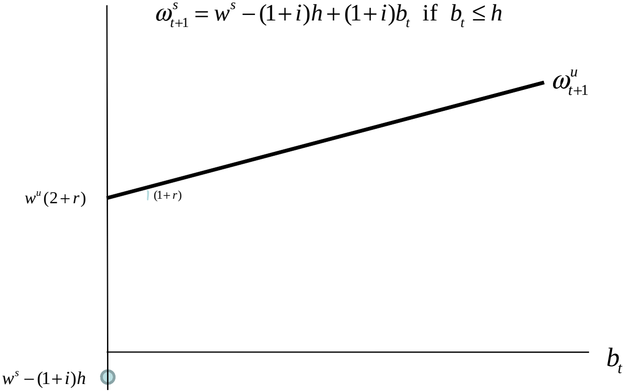

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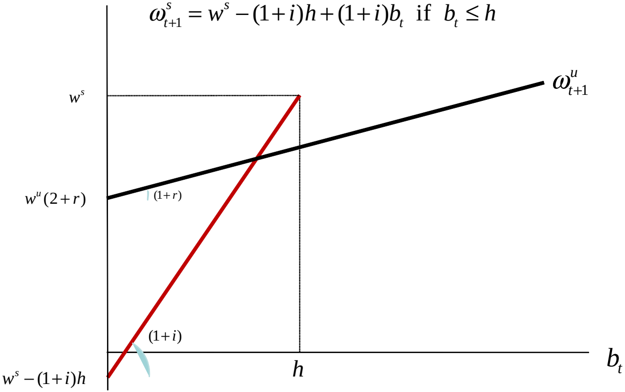

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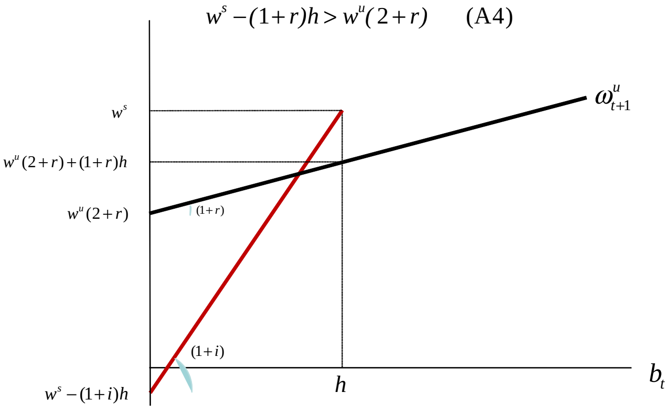
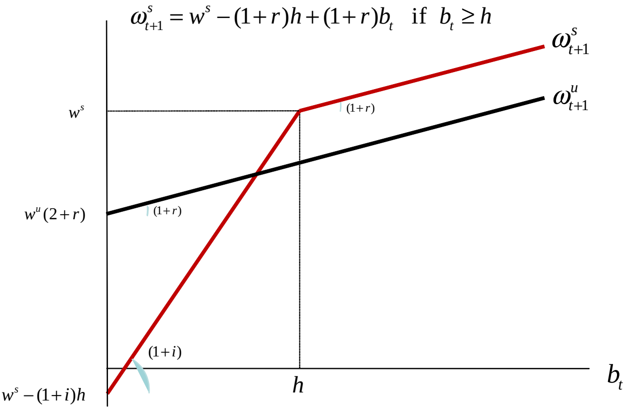
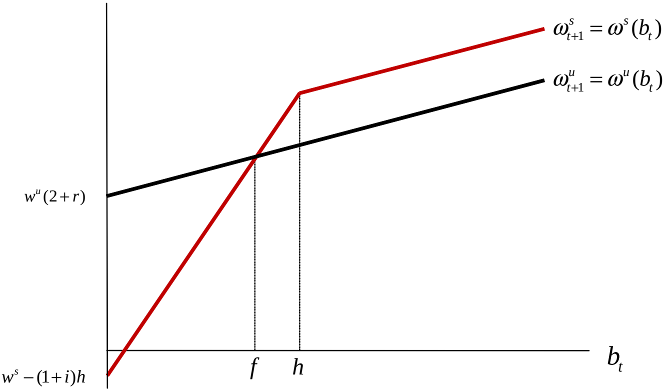


Income from Being Skilled Worker: Lenders



Bequest and Occupational Choice



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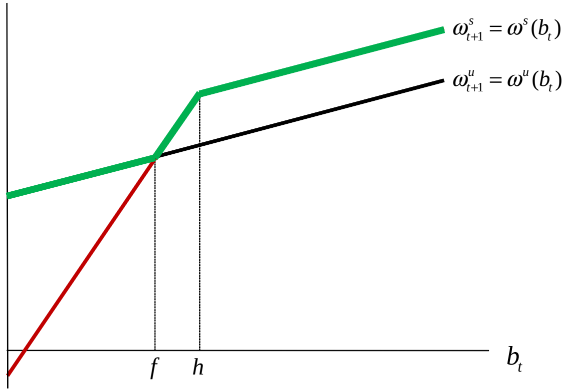

Bequest and Occupational Choice

$$b_t \quad \begin{cases} < f & \rightarrow \omega^u_{t+1} > \omega^s_{t+1} \text{ (individual } t \text{ becomes unskilled)} \\ > f & \rightarrow \omega^u_{t+1} < \omega^s_{t+1} \text{ (individual } t \text{ becomes skilled)} \end{cases}$$

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where

$$f = \frac{w^u(2+r) - [w^s - (1+i)h]}{i - r} > 0$$

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b_{t+1}

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Bequest Dynamics: Sufficiet Conditions for Multiplicity of Steady-Sate

$$\begin{aligned} (1-\alpha)(1+r) &< 1 \\ (1-\alpha)(1+i) &> 1 \end{aligned} \tag{A5}$$

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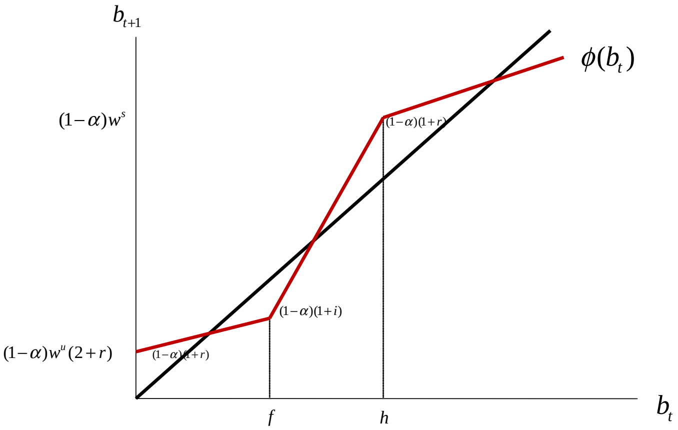
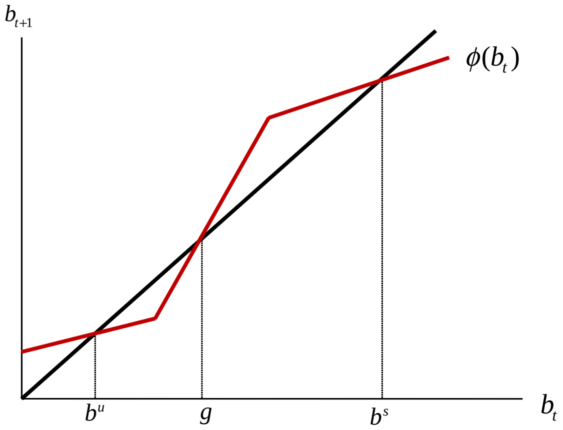
$$(1-\alpha)w^s > h \tag{A6}$$

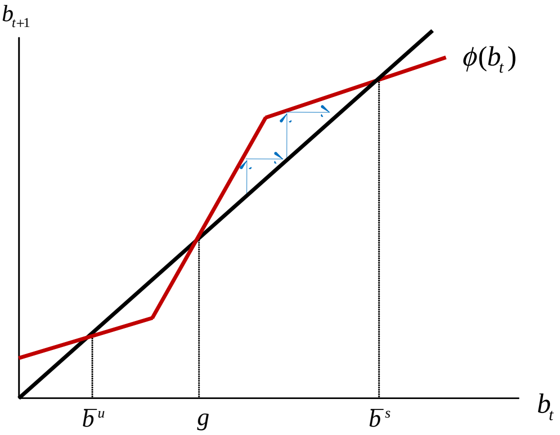
Bequest Dynamics



Bequest Dynamics: Multiple Steady-State Equilibrium



Bequest Dynamics: Stability of High Bequest Equilibrium



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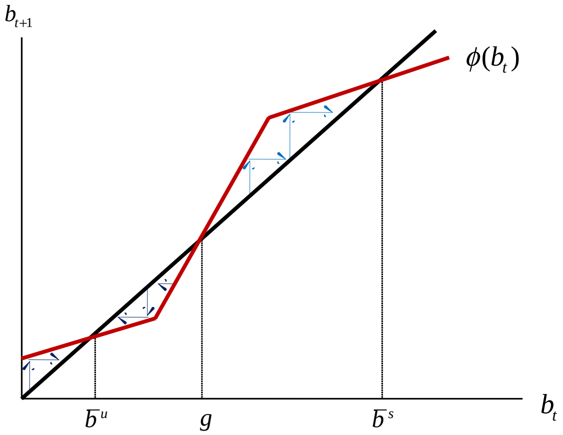
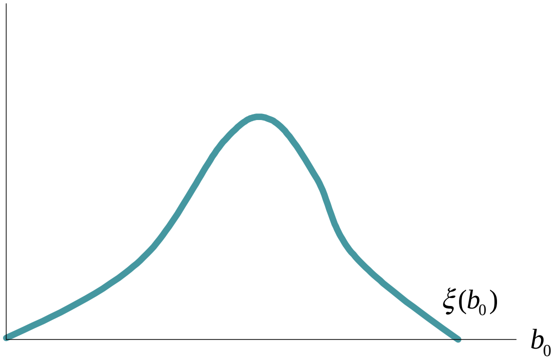


The Distribution of the Inheritance in Period t



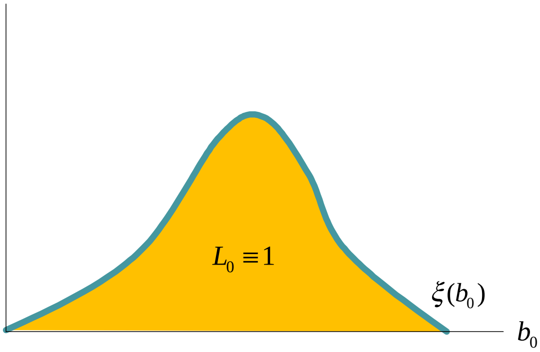
Income Distribution and the Long Run Decomposition of the Labor Force

$\xi_t(b_t) \equiv$ Distribution of inheritance at time t

$\Longrightarrow$

$$L_t = \int_0^{\infty} \xi(b_t) db_t \equiv 1$$

The Distribution of the Inheritance in Period t



Income Distribution of the Long Run Decomposition of the Labor Force

$$\lim_{t\to\infty} l_t^u = \int_0^g \xi_t(b_t)db_t \equiv \bar{l}^u$$

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where

$$\partial \bar{l}^s / \partial g < 0$$

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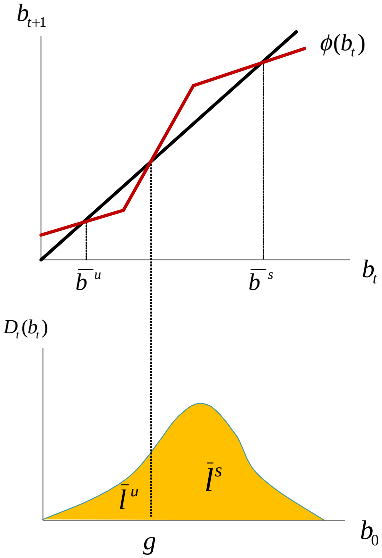
where

$$\partial\bar{l}^s/\partial g < 0$$

and

$$g = \frac{(1-\alpha)[(1+i)h - w^s]}{(1-\alpha)(1+i)-1} > 0$$

Income Distribution of Skill Composition



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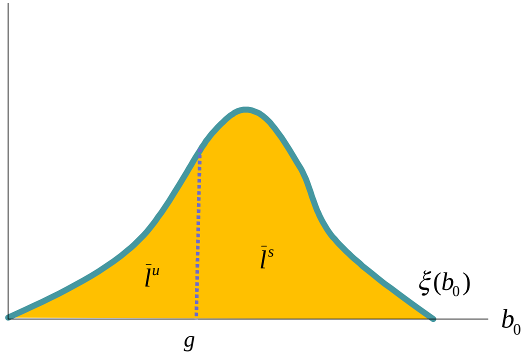
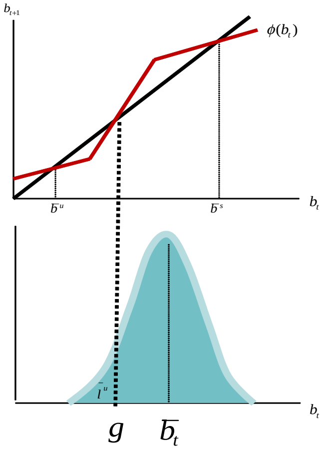
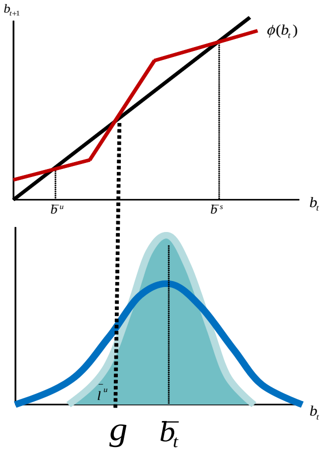


Inequality and Development: Rich Economies



Rich Economies: Inequality is Harmful for Development

Inequality reduces human capital formation



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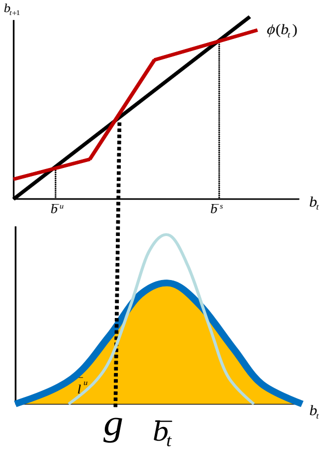
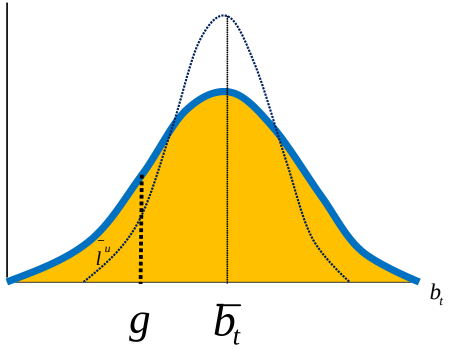
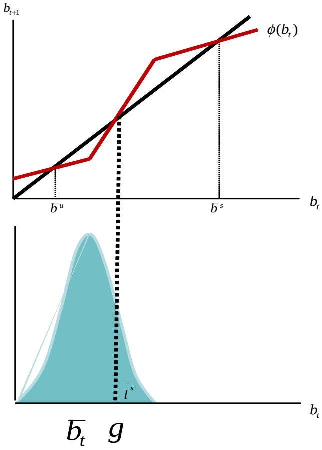


Rich Economies: Inequality is Harmful for Development

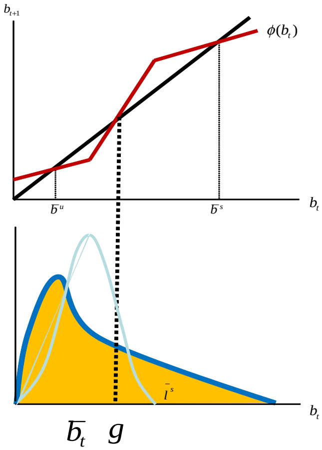


Inequality and Development: Poor Economies



Poor Economies: Inequality may Benefit Development

Inequality stimulates human capital formation



Poor Economies: Inequality may Benefit Development